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Comparison of Standard and Heat-Pipe Fins with Specified Tip Temperature Condition

W. Jerry Bowman* and Daniel Maynes†
Brigham Young University, Provo, Utah 84602

Analytical expressions for heat-pipe fin temperature distribution and efficiency were obtained for a fin with uniform cross-sectional area, convection, thermal conductivity, and where the fin-tip temperature is specified. The results were compared to a standard (solid conductor) fin. It was observed that under most conditions, the heat-pipe fin would have a higher efficiency than a geometrically similar standard fin. The important parameters needed to compare the two types of fins are presented.

Nomenclature

A	=	fin cross-sectional area
A_v	=	vapor space cross-sectional area
a	=	nondimensional fin parameter
B	=	$n^2\theta_v/(m^2+n^2)$
h_i	=	convection coefficient, inside of fin
h_o	=	convection coefficient, outside of fin
k	=	thermal conductivity
k_{eff}	=	effective thermal conductivity of wick
L	=	fin length
M	=	nondimensional fin parameter
m	=	nondimensional fin parameter
N	=	nondimensional fin parameter
n	=	nondimensional fin parameter
P_i	=	inside perimeter of fin
P_o	=	outside perimeter of fin
T	=	temperature
T_b	=	fin-base temperature
T_v	=	vapor space temperature
T_∞	=	fin environment temperature
t	=	thickness of heat-pipe wick
X	=	nondimensional axial location
x	=	axial location
Z	=	$a(m^2+n^2)^{1/2}$
η	=	efficiency
η_f	=	standard fin efficiency
$\eta_{\text{HP},f}$	=	heat-pipe fin efficiency
θ	=	nondimensional temperature
θ_L	=	nondimensional fin-tip temperature
θ_v	=	nondimensional vapor temperature

Introduction

FINS are added to objects to increase surface area and thus heat transfer. One possible method of increasing fin efficiency is to include a heat pipe in the fin. The high effective thermal conductivity of the heat pipe will aid energy transport from the base of the fin to its tip. Bowman et al.[1] derived a useful expression for the efficiency of a heat-pipe fin with an insulated fin-tip condition. Bowman et al.[2] expanded the results to include the case where the fin is inserted into the object to be cooled. Using the expressions, it is possible to compare the efficiency of a heat-pipe fin to a standard fin. This is useful in the early phase of a cooling system design. It allows one to determine if heat-pipe fins might outperform standard fins before having to invest time and money in the heat-pipe fin design. Both of the earlier works assumed a constant-area, constant thermal conductivity heat-pipe fin with uniform inside and outside convection and an adiabatic fin-tip condition. The purpose of this work is to study a similar heat-pipe fin, except with a specified tip temperature boundary condition. As was done before, the results will be compared to standard fin results. As was observed in the earlier work, adding a heat pipe to the fin does not always improve the efficiency of the fin. For some conditions the standard fin will outperform the heat-pipe fin. The important parameters required to compare the different fins are defined next.

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*Associate Professor, Department of Mechanical Engineering. Senior Member AIAA.
†Assistant Professor, Department of Mechanical Engineering.

Heat-Pipe Fin Efficiency

Figure 1 illustrates the heat-pipe fin being considered. Neglecting radiation, assuming steady state, and assuming temperature varies only in the x direction, conservation of energy applied to a differential element of the wall is

$$k(A-A_v)\frac{\partial^2 T}{\partial x^2} - h_o P_o(T-T_\infty) - h_i P_i(T-T_v) = 0 \tag{1}$$

The nondimensional temperature and length can be defined as

$$\theta = (T-T_\infty)/(T_b-T_\infty) \tag{2}$$

$$X = (x)/(L) \tag{3}$$

Rewriting Eq. (1) in terms of the nondimensional variables yields

$$\frac{\partial^2\theta}{\partial X^2} - a^2(m^2+n^2)\theta = -a^2n^2\theta_v \tag{4}$$

where

$$m^2 = h_o P_o L^2 \big/ kA \tag{5}$$

$$n^2 = h_i P_i L^2 \big/ kA \tag{6}$$

$$a^2 = A/(A-A_v) \tag{7}$$

$$\theta_v = (T_v-T_\infty)/(T_b-T_\infty) \tag{8}$$

The parameters a^2, n^2, and m^2 were carefully selected to make it easy to compare the heat-pipe fin's performance with a standard fin; m^2 as defined in Eq. (5) is a standard parameter used in fin design. It is the ratio of conduction resistance L/kA to convection resistance $1/h_oP_oL$ for the standard fin. A similar parameter that is important in heat-pipe fins only is n^2. It is the ratio of conduction resistance along the fin to convection resistance associated with the heat-pipe wick and evaporation on the inside of the heat-pipe fin $1/h_iP_iL$. The last parameter a^2 is the ratio of the total fin cross-sectional area A to the heat-pipe wall area $A-A_v$. Realistic values for these three parameters and their importance were discussed by Bowman et al.[1]

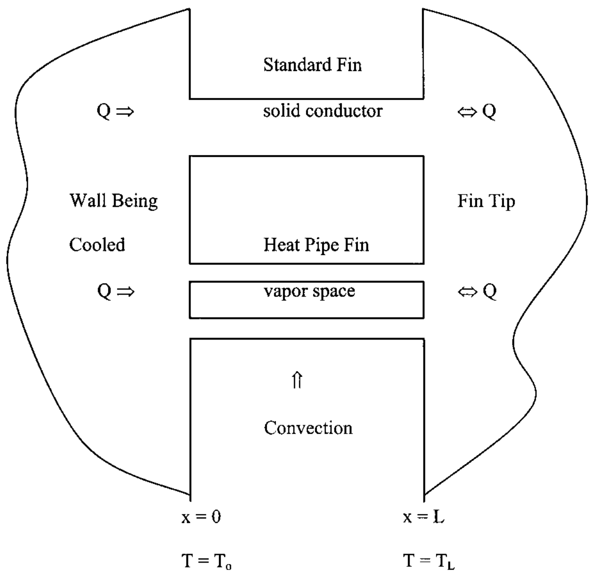


Fig. 1 Standard and heat-pipe fins with specified end temperature conditions.

It was assumed that the convection coefficient inside the heat pipe was uniform along the heat pipe. This is often a reasonable assumption because energy transport from the wall to the vapor occurs through the liquid saturated wick and then from the wick surface to the vapor. This energy transport can be represented using an electrical analogy as energy transport through two thermal resistors. The thermal resistance of the wick is typically much larger than the thermal resistance caused by evaporation or condensation.[3] Thus, h_i can be approximated by examining conduction through the wick, or

$$h_i = k_{\mathrm{eff}}/t \tag{9}$$

Assuming the liquid is uniformly distributed along a wick of uniform thickness, this thermal resistance will also be uniform along the length of the heat pipe.

Equation (4) can be solved for $\theta(X)$. Assuming T_∞, T_v, h_i, and h_o are constants, the solution is

$$\theta(X) = c_1[\sinh(ZX) + \cosh(ZX)] + c_2[\cosh(ZX) - \sinh(ZX)] + \frac{n^2\theta_v}{(m^2 + n^2)} \tag{10}$$

where $Z = a(m^2 + n^2)^{1/2}$. The constants c_1 and c_2 are evaluated by applying the appropriate boundary conditions. Assuming the base temperature is known and the fin-tip temperature is specified, the boundary conditions are

$$\theta(0) = 1.0 \tag{11}$$

$$\theta(1) = \theta_L = \frac{T(L) - T_\infty}{T_b - T_\infty} \tag{12}$$

The constants c_1 and c_2 are

$$c_1 = \frac{(1 - B)[\sinh(Z) - \cosh(Z)] - B + \theta_L}{2\sinh(Z)}$$

$$c_2 = \frac{(1 - B)[\sinh(Z) + \cosh(Z)] + B - \theta_L}{2\sinh(Z)} \tag{13}$$

where

$$B = \frac{n^2\theta_v}{(m^2 + n^2)}$$

The model assumes the vapor temperature θ_v is constant along the heat pipe. This temperature will adjust until evaporation matches condensation in the heat pipe. Conservation of energy for the vapor space is

$$\int_0^L h_i P_i (T - T_v)\,\mathrm{d}x = 0 \qquad \text{or} \qquad \int_0^1 \theta\,\mathrm{d}x = \theta_v \tag{14}$$

Substituting Eqs. (10) and (13) into Eq. (14), θ_v can be found to be

$$\theta_v = \frac{(\theta_L + 1)[\cosh(Z) - 1]}{Z\sinh(Z) + (a^2n^2/Z^2)[2\cosh(Z) - Z\sinh(Z) - 2]} \tag{15}$$

An important fin parameter is the fin efficiency. It is defined as the heat transfer from the object divided by the maximum heat transfer if the entire fin is maintained at the fin's base temperature. Often, the energy leaving the object is found by integrating the convection from the fin. Conduction through the tip of the fin must also be considered. For the specified tip temperature fin the energy leaving the object is more than the convection from the fin. For this reason the energy leaving the object must be found by evaluating the conduction into the base of the fin. The efficiency can be found from the expression

$$\eta = \frac{-kA(\mathrm{d}T/\mathrm{d}x_{x=0})}{h_o P_o L(T_b - T_\infty)} = -\frac{1}{a^2m^2}\frac{\mathrm{d}\theta}{\mathrm{d}X_{X=0}} \tag{16}$$

Evaluating Eq. (16) for the case of the heat-pipe fin gives

$$\eta_{\mathrm{HP},f} = \frac{Z}{a^2m^2}\left[\frac{(1 - B)\cosh(Z) + B - \theta_L}{\sinh(Z)}\right] \tag{17}$$

The heat-pipe fin efficiency can be compared to the efficiency of a standard, constant-area fin with the same boundary conditions and geometry. Using results from Ref. 4, the efficiency of the standard fin can be shown to be

$$\eta_f = \frac{\cosh m - \theta_L}{m\sinh m} \tag{18}$$

Results

The analytical results will be studied next. Three sets of graphs will be presented. First, the axial temperature distribution for standard and heat-pipe fins will be compared. Second, the efficiency of the heat-pipe fins and the standard fins will be compared. Lastly, a set of design curves that pertain only to heat-pipe fins and allow for the easy determination of a heat-pipe fin's efficiency are presented.

Figures 2 and 3 are graphs of Eq. (10) and the temperature distribution for a standard fin.[3] In both figures the area ratio a is 1.25. Recall that the area ratio compares the total cross-sectional area of the fin to the cross-sectional area of the heat-pipe fin wall. A value of

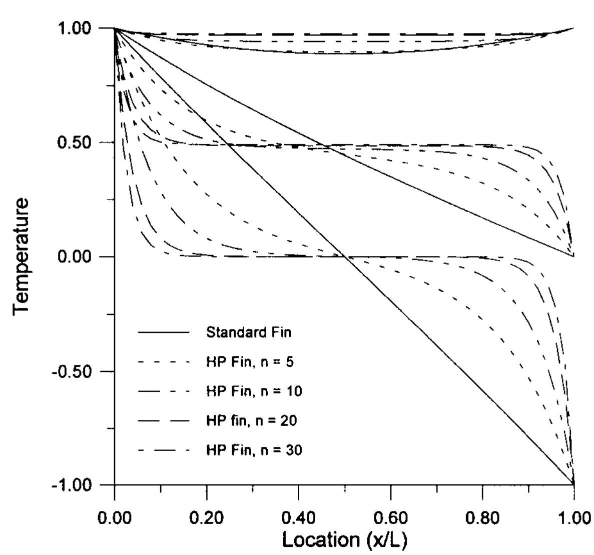


Fig. 2 Temperature distribution, (a = 1.25, m = 1).

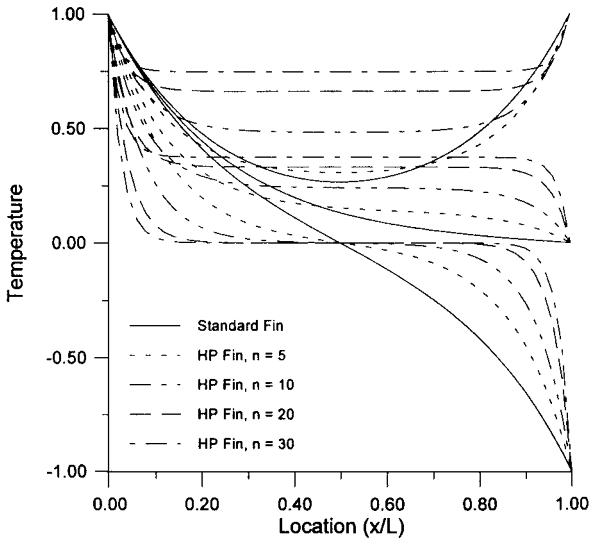
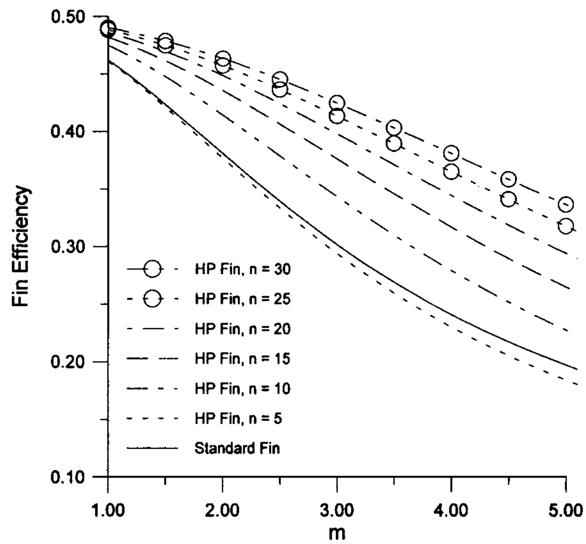


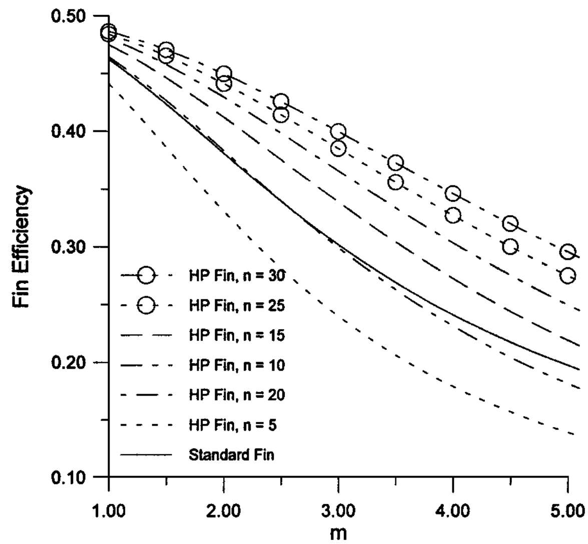
Fig. 3 Temperature distribution, ($a = 1.25, m = 4$).

1.25 was selected for illustration only. Figure 2 assumes the parameter m is equal to one, whereas Fig. 3 assumes it is equal to four. The parameter m is the ratio of resistance to axial conduction along the fin to convection resistance from the fin. The small value for this ratio corresponds to a shorter fin, a fin with a high thermal conductivity, or one with poor convection. The larger value corresponds to a longer fin, one with low thermal conductivity or one with good convection. Each figure contains three sets of five curves. The three sets correspond to three different fin-tip boundary conditions. For the upper set of curves, both ends of the fin are at the same temperature ($\theta = 1.0$). For the other sets of curves, the nondimensional tip temperatures are 0.5, 0.0, −0.5, and −1.0. As an example, the case where $\theta = 0.0$ corresponds to the fin-tip temperature being equal to the surrounding environment temperature.

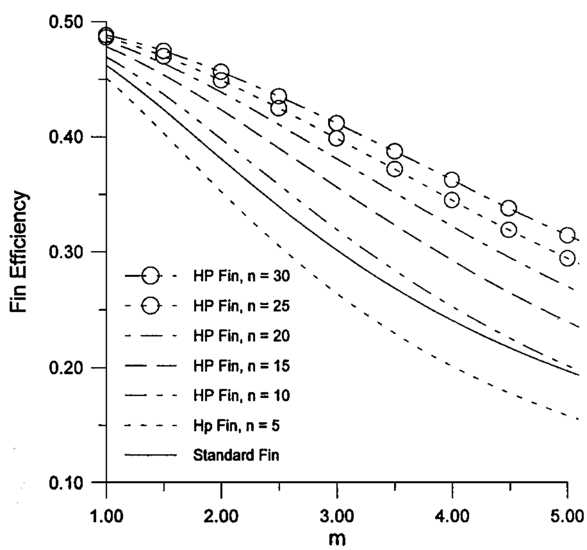
For each set of boundary conditions, five examples are shown. They compare the temperature distributions of a standard fin and four different heat-pipe fins. The different heat-pipe fins have different values of the n parameter. Recall that n is the ratio of axial conduction along the heat pipe to internal resistance between the heat-pipe wall and the vapor space. Small values of n correspond to shorter fins, high thermal conductivity fins, or one with large thermal resistance between the wall and the vapor space.



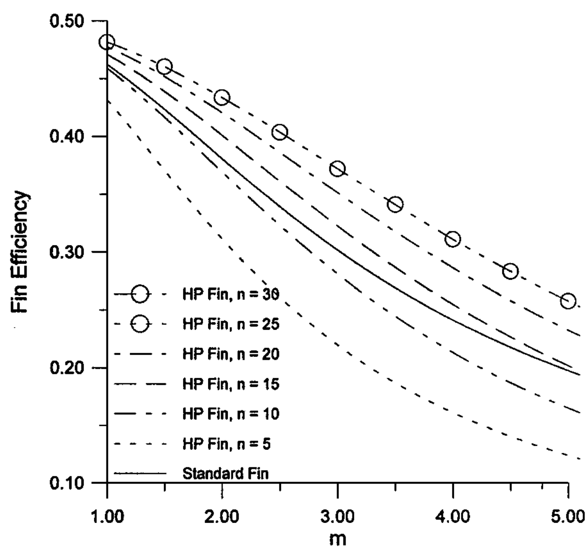
a) $a = 1.25$ and $\theta_L = 1.0$



c) $a = 1.75$ and $\theta_L = 1.0$

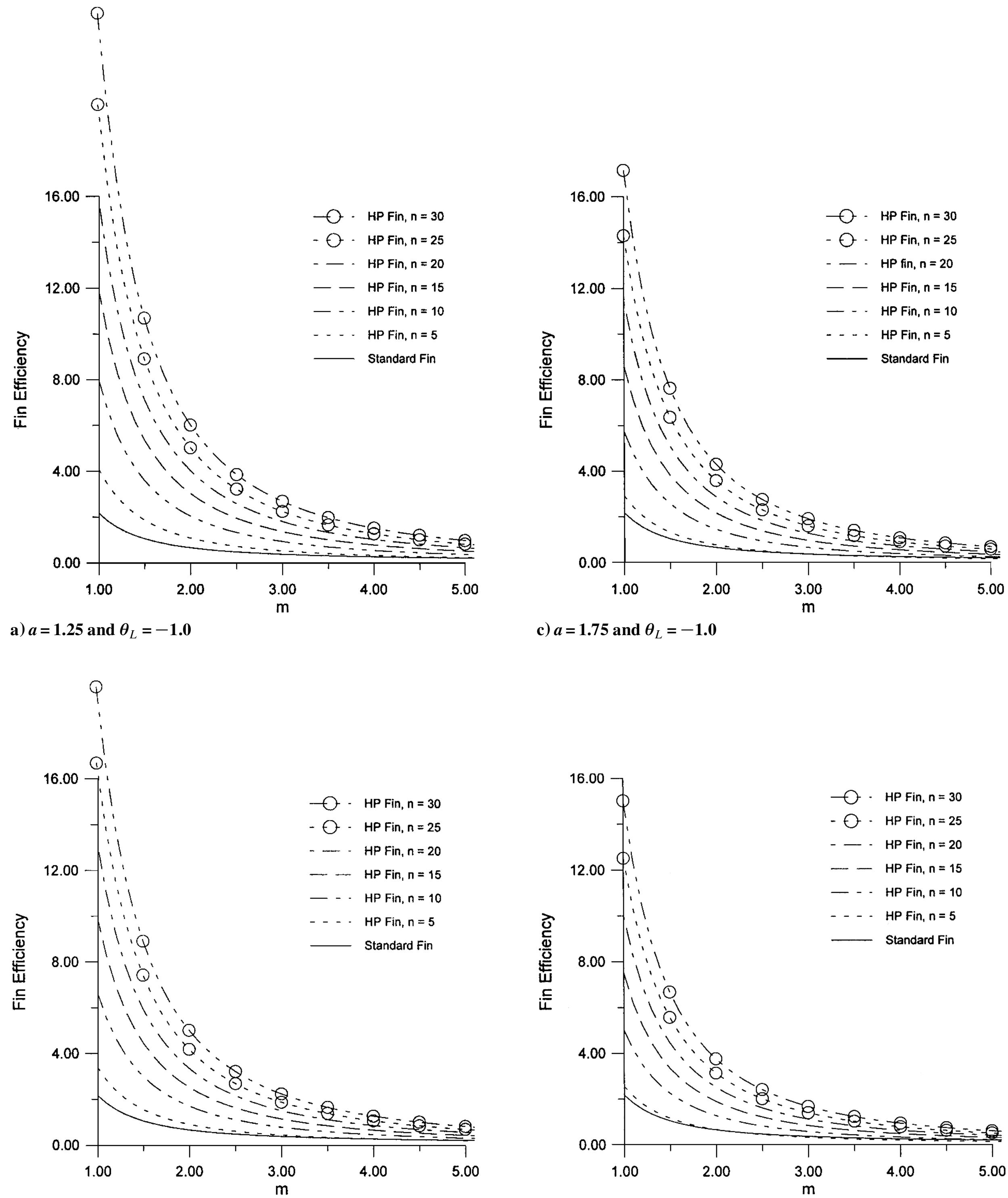


b) $a = 1.5$ and $\theta_L = 1.0$



d) $a = 2.0$ and $\theta_L = 1.0$

Fig. 4 Fin efficiency for various area ratios and $\theta_L = 1.0$.



a) $a = 1.25$ **and** $\theta_L = -1.0$

b) $a = 1.5$ **and** $\theta_L = -1.0$

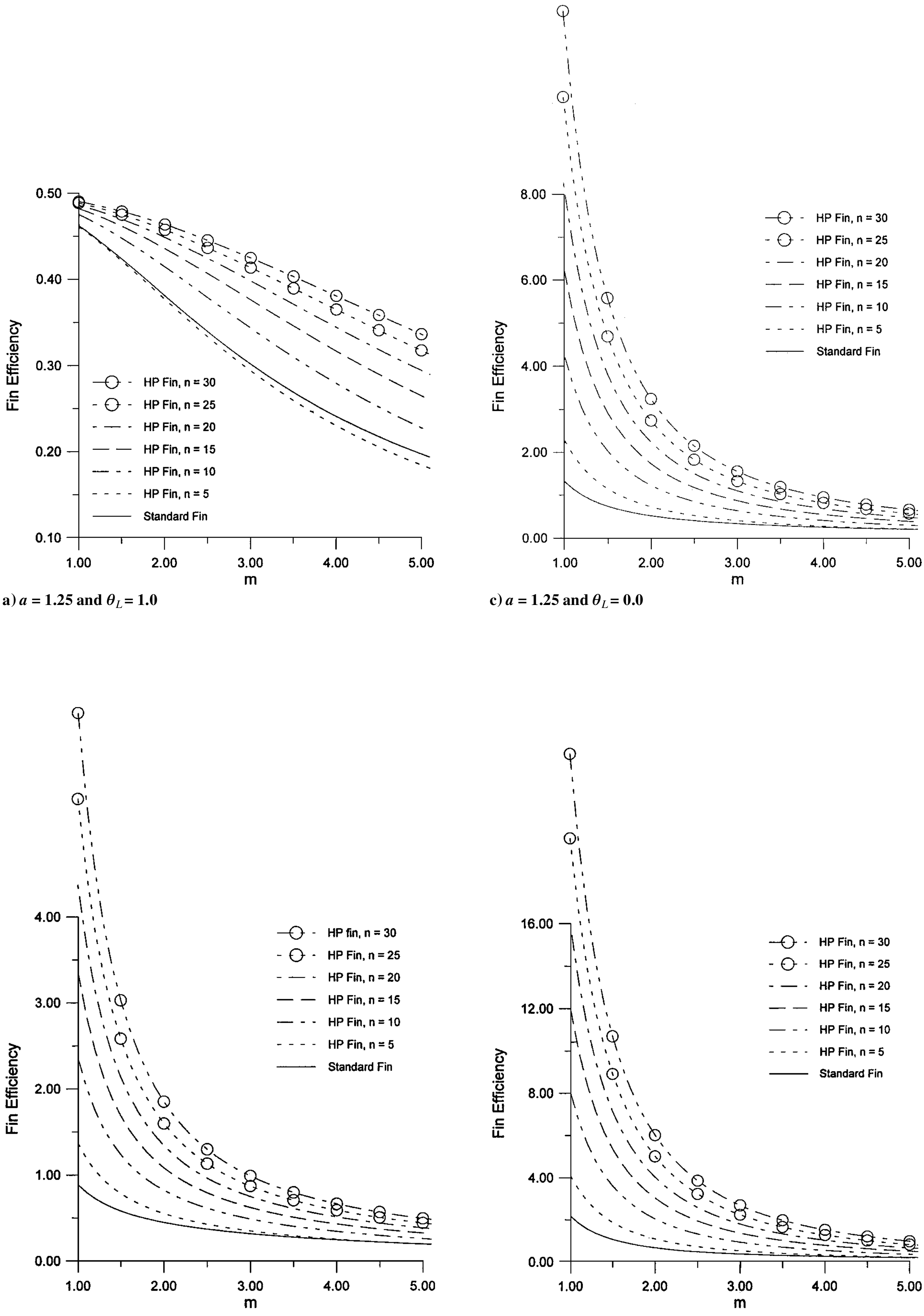
c) $a = 1.75$ **and** $\theta_L = -1.0$

d) $a = 2.0$ **and** $\theta_L = -1.0$

Fig. 5 Fin efficiency for various area ratios and $\theta_L = -1.0$**.**

Several observations can be made from the curves. In comparing the standard fin temperature distributions, for smaller values of m the temperature distributions are more linear. Fins with larger values of m are influenced more by convection from the fin caused by increased fin length, increased convection heat-transfer coefficient, or low fin thermal conductivity. From the figures it can be seen that as n increases the variation in temperature distribution from the standard fin also increases. Later we show that as n increases the heat-pipe fin will typically have a higher efficiency than the standard fin. A last observation is the flatness of the heat-pipe fin temperature distributions in the fin midsection. This indicates large axial conduction on the ends of the fin and small axial conduction in its midsection. In contrast, the standard fin demonstrates axial conduction along its entire length. For the heat-pipe fin case energy is transported through the midsection via the vapor motion in the heat pipe.

From a design perspective Figs. 4–6 present the most useful results of this research. They compare the standard fin efficiency to the heat-pipe fin efficiency for different problem parameters. Figures 4 and 5 illustrate the effects of m, n, and a on fin performance.



a) $a = 1.25$ and $\theta_L = 1.0$

c) $a = 1.25$ and $\theta_L = 0.0$

b) $a = 1.25$ and $\theta_L = 0.5$

d) $a = 1.25$ and $\theta_L = -1.0$

Fig. 6 Fin efficiency for various tip conditions and $a = 1.25$.

Recall that a is the ratio of the standard fin cross-sectional area to the conduction area for the heat-pipe fin. Figure 6 illustrates the effects of m, n, and fin-tip temperature θ_L on fin performance.

Figure 4 illustrates the influence of area ratio on fin efficiency. With a equal to 1.25, most heat-pipe fins will outperform the standard fins. Only the heat-pipe fins with n less than five are inferior to the standard fins (Fig. 4a). As the area ratio increases to 2.0, n must be higher than 15 for the heat-pipe fin to outperform the standard fin (Fig. 4d). Thus, the smaller the area ratio, the better the heat-pipe fin is. Physically, a small value of area ratio corresponds to a thick walled heat-pipe fin. This may seem counterintuitive to some heat-pipe designers. These results exist because of an earlier assumption that the heat pipe is flush mounted to the object. Consequently, the only way energy can enter the heat pipe is via axial conduction along the heat-pipe wall. The thicker-walled heat pipes outperform the thin-walled heat pipes, because it provides for increased axial conduction from a larger conduction area. The area ratio is less important of a parameter if the flush-mounted assumption is not made and a better conduction path to the heat pipe evaporator is provided.[2]

Figure 5 is similar to Fig. 4 except the fin-tip temperature has been changed. In Fig. 4 θ_L was 1.0 (both ends of the fin are at the same temperature), whereas in Fig. 5 it is -1.0 (the fin-end temperatures are equal amounts above and below the environment temperature). In contrast to Fig. 4, Fig. 5 illustrates how the different fin-tip boundary condition effects performance. The efficiencies in Fig. 5 are generally an order of magnitude higher than those shown in Fig. 4. This is caused by the increase in axial conduction that occurs in the fins shown in Fig. 5. Energy leaving the object is convected away from the fin as well as conducted along the fin to the fin tip. Recall, the definition of efficiency used earlier compared the total energy leaving the object via the fin to the maximum energy that could be convected from the fin [Eq. (16)]. This definition allows efficiency to be greater than 1.0 if a large amount of energy is conducted out of the fin tip.

Other observations can be made from Fig. 5. The influence of m on efficiency can be seen. For small values of m, the heat-pipe fin significantly outperforms the standard fin. As m increases, the difference in performance is decreased. As noticed earlier, larger values of n show an increase in efficiency for the heat-pipe fins.

Figure 6 is included to illustrate the effect of fin-tip temperature on fin performance. As the fin-tip temperature decreases, more energy is removed from the object caused by conduction along the fin. This conduction added to the convection from the fin increases the total energy transported by the fin and thus increases the fin efficiency. Decreasing the tip temperature has a larger impact on the heat-pipe fin than the standard fin.

Conclusions

Analytical expressions for heat-pipe fin temperature distribution and efficiency were obtained. Fin cross-sectional area, external convection, internal convection, and thermal conductivity were assumed to be constant. The tip temperature of the fin was specified. The heat-pipe fin results were compared to standard fin results. It was observed that under most conditions the heat-pipe fin will have a higher efficiency than a standard fin.

The results presented in this paper are useful in the preliminary design of a project before a detailed heat-pipe design is conducted. They make it possible to compare standard and heat-pipe fins early in the design. If the preceding results indicate that a heat-pipe fin should be superior to a standard fin, the designer can then continue with the heat pipe design process. This will include testing to ensure that the well-known heat-pipe thermal limits will not be violated.

References

[1] Bowman, W. J., Storey, J. K., and Svensson, K. I., "Analytical Comparison of Constant Area, Adiabatic Tip, Standard Fins and Heat Pipe Fins," *Journal of Thermophysics and Heat Transfer*, Vol. 13, No. 2, 1999, pp. 269–272.

[2] Bowman, W. J., Moss, T. W., Maynes, D., and Paulson, K. A., "Efficiency of a Constant Area, Adiabatic Tip, Heat-Pipe Fin," *Journal of Thermophysics and Heat Transfer*, Vol. 14, No. 1, 2000, pp 112–114.

[3] Dunn, P., and Reay, D. A., *Heat Pipes*, 2nd ed., Pergamon, Oxford, England, U.K., 1978, pp. 80–82.

[4] Incropera, F. P., and DeWitt, D. P., *Fundamentals of Heat and Mass Transfer*, 4th ed., Wiley, 1996, pp. 117, 118.